APPENDIX

Introduction to General Equilibrium Theory

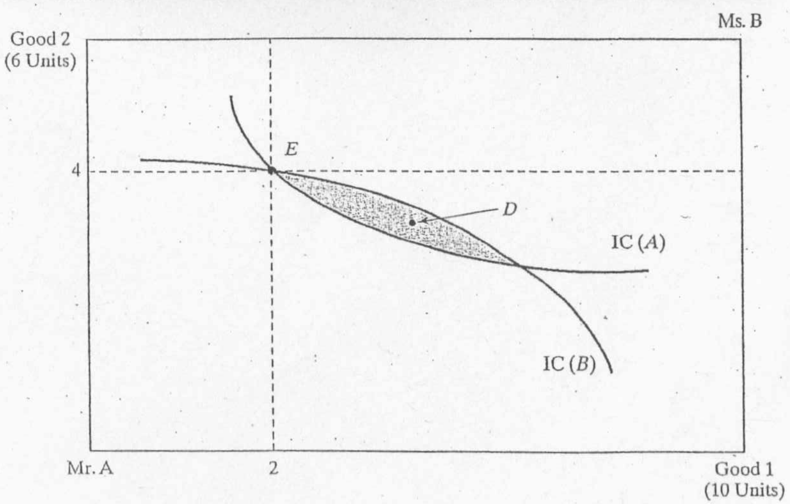
The goal of this appendix is to provide an introduction to the essentials of General Equilibrium Theory thereby permitting a complete understanding of Section 1.6 of the present chapter and facilitating the discussion of subsequent chapters (from Chapter 7 on). To make this presentation as simple as possible we'll take the case of a hypothetical exchange economy (that is, one with no production) with two goods and two agents. This permits using a very useful pedagogical tool known as the Edgeworth-Bowley box.

Let us analyze the problem of allocating efficiently a given economy-wide endowment of 10 units of good 1 and 6 units of good 2 among two agents, *A* and *B*. In Figure A1-1, we measure good 2 on the vertical axis and good 1 on the horizontal axis. Consider the choice problem from the origin of the axes for Mr. A, and upside down (that is, placing the origin in the upper right corner), for Ms. B. An allocation is then represented as a point in a rectangle of size 6 × 10. Point *E* is an allocation at which Mr. A receives 4 units of good 1 and 2 units of good 2. Ms. B gets the rest, that is, 2 units of good 1 and 8 units of good 2. All other points in the box represent feasible allocations, that is, alternative ways of allocating the resources available in this economy.

PARETO OPTIMAL ALLOCATIONS

In order to discuss the notion of Pareto optimal or efficient allocations, we need to introduce agents' preferences. They are fully summarized, in the graphical context of the Edgeworth-Bowley box, by indifference curves (IC) or utility level curves. Thus, starting from the allocation *E* represented in Figure A1-1, we can record all feasible allocations that provide the same utility to Mr. A. Exactly how such a level curve looks is person specific, but we can be sure that it slopes downward. If we take away some units of good 2, we have to compensate him with some extra units of good 1 if we are to

FIGURE A1-1 The Edgeworth-Bowley Box: The Set of Pareto Superior Allocations



leave his utility level unchanged. It is easy to see as well that the ICs of a consistent person do not cross, a property associated with the notion of transitivity (and with rationality) in our next chapter. And we have seen in Boxes 1-1 and 1-2 that the preference for smoothness translates into convex-to-the-origin level curves as drawn in Figure A1-1. The same properties apply to the IC of Ms. B, of course viewed upside down with the upper right corner as the origin.

With this simple apparatus we are in a position to discuss further the concept of Pareto optimality. Arbitrarily tracing the level curves of Mr. A and Ms. B as they pass through allocation *E* (but in conformity with the properties derived in the previous paragraph), only two possibilities may arise: they cross each other at *E* or they are tangent to one another at point *E*. The first possibility is illustrated in Figure A1-1, the second in Figure A1-2. In the first case, allocation *E* cannot be a Pareto optimal allocation. As the picture illustrates clearly, by the very definition of level curves, if the ICs of our two agents cross at point *E* there is a set of allocations (corresponding to the shaded area in Figure A1-1) that are simultaneously preferred to *E* by both Mr. A and Ms. B. These allocations are Pareto superior to *E*, and, in that situation, it would indeed be socially inefficient or wasteful to distribute the available resources as indicated by *E*. Allocation *D*, for instance, is feasible and preferred to *E* by both individuals.

To the contrary, if the ICs are tangent to one another at point *E'* as in Figure A1-2, no redistribution of the given resources exists that would be approved by both agents. Inevitably, moving away from *E'* decreases the utility level of one of the two agents if it favors the other. In this case, *E'* is a Pareto optimal allocation. Figure A1-2 illustrates that it is not generally unique, however. If we connect all the points where the various ICs of our two agents are tangent to each other, we draw the line, labeled the contract curve, representing the infinity of Pareto optimal allocations in this simple economy.

An indifference curve for Mr. A is defined as the set of allocations that provide the same utility to Mr. A as some specific allocation; for example, allocation *E*:

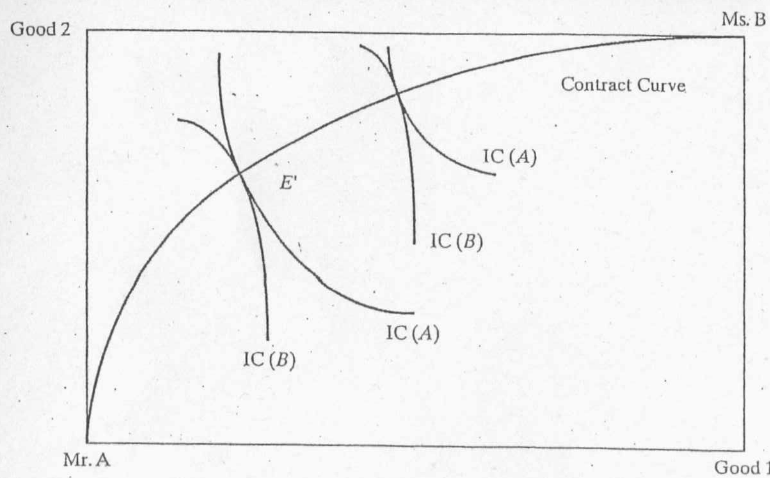
$$\{(c_1^A, c_2^A) : U(c_1^A, c_2^A) = U(E)\}.$$

This definition implies that the slope of the IC can be derived by taking the total differential of $U(c_1^A, c_2^A)$ and equating it to zero (no change in utility along the IC), which gives:

$$\frac{\partial U(c_1^A, c_2^A)}{\partial c_1^A} dc_1^A + \frac{\partial U(c_1^A, c_2^A)}{\partial c_2^A} dc_2^A = 0, \quad \textbf{(A1.1)}$$

and thus,

FIGURE A1-2 The Edgeworth-Bowley Box: The Contract Curve



$$\frac{dc_2^A}{dc_1^A} = -\frac{\dfrac{\partial U(c_1^A, c_2^A)}{\partial c_1^A}}{\dfrac{\partial U(c_1^A, c_2^A)}{\partial c_2^A}} \equiv -\text{MRS}_{1,2}^A. \qquad \text{(A1.2)}$$

That is, the negative (or the absolute value) of the slope of the IC is the ratio of the marginal utility of good 1 to the marginal utility of good 2, specific to Mr. A and to the allocation (c_1^A, c_2^A) at which the derivatives are taken, which defines Mr. A's Marginal Rate of Substitution (MRS) between the two goods.

Equation (A1.2) permits a formal characterization of a Pareto optimal allocation. Our former discussion has equated Pareto optimality with the tangency of the ICs of Mr. A and Ms. B. Tangency, in turn, means that the slopes of the respective ICs are identical. Allocation E, associated with the consumption vector $(c_1^A, c_2^A)^E$ for Mr. A and $(c_1^A, c_2^A)^E$ for Ms. B, is thus Pareto optimal if and only if

$$\text{MRS}_{1,2}^A = \frac{\dfrac{\partial U(c_1^A, c_2^A)^E}{\partial c_1^A}}{\dfrac{\partial U(c_1^A, c_2^A)^E}{\partial c_2^A}} = \frac{\dfrac{\partial U(c_1^B, c_2^B)^E}{\partial c_1^B}}{\dfrac{\partial U(c_1^B, c_2^B)^E}{\partial c_2^B}} \equiv \text{MRS}_{1,2}^B. \qquad \text{(A1.3)}$$

Equation (A1.3) provides a complete characterization of a Pareto optimal allocation in an exchange economy except in the case of a corner allocation, that is, an allocation at the frontier of the box where one of the agents receives the entire endowment of one good and the other agent receives none. In that situation it may well be that the equality could not be satisfied except, hypothetically, by moving to the outside of the box, that is, to allocations that are not feasible since they require giving a negative amount of one good to one of the two agents.

So far we have not touched on the issue of how the discussed allocations may be determined. This is the viewpoint of Pareto optimality analysis exclusively concerned with deriving efficiency properties of given allocations, irrespective of how they were achieved. Let us now turn to the concept of competitive equilibrium.

COMPETITIVE EQUILIBRIUM

Associated with the notion of competitive equilibrium is the notion of markets and prices. One price vector one price for each of our two goods, or simply a relative price taking good 1 as the numeraire, and setting $p_1 = 1$, is represented in the Edgeworth-Bowley box by a downward sloping line. From the viewpoint of either agent, such a line has all the properties of the budget line. It also represents the frontier of their opportunity set. Let us assume that the initial allocation, before any trade, is represented by point I in Figure A1-3. Any line sloping downward from I does represent the set of allocations that Mr. A, endowed with I, can obtain by going to the market and exchanging (competitively, taking prices as given) good 1 for 2 or vice versa. He will maximize his utility subject to this budget constraint by attempting to climb to the highest IC making contact with his budget set. This will lead him to select the allocation corresponding to the tangency point between one of his ICs and the price line. Because the same prices are valid for both agents, an identical procedure, viewed upside down from the upper right-hand corner of the box, will lead Ms. B to a tangency point between one of her ICs and the price line. At this stage, only two possibilities may arise: Mr. A and Ms. B have converged to the same allocation (the two markets, for goods 1 and 2, clear—supply and demand for the two goods are equal and we are at a competitive equilibrium); or the two agents' separate optimizing procedures have lead them to select two different allocations. Total demand does not equal total supply and an equilibrium is not achieved. The two situations are described, respectively, in Figures A1-3 and A1-4.

In the disequilibrium case of Figure A1-4, prices will have to adjust until an equilibrium is found. Specifically, with Mr. A at point A and Ms. B at point B, there is an excess demand of good 2 but insufficient demand for good 1. One would expect the price of 2 to increase relative to the price of good 1 with the likely result that both agents will decrease their net demand for 2 and increase their net demand for 1. Graphically, this is depicted by the price curve tilting with point I as the axis and looking less steep (indicating, for instance, that if both agents wanted to buy good 1 only, they could now afford more of it). With regular ICs, the respective points of tangencies will converge until an equilibrium similar to the one described in Figure A1-3 is reached.

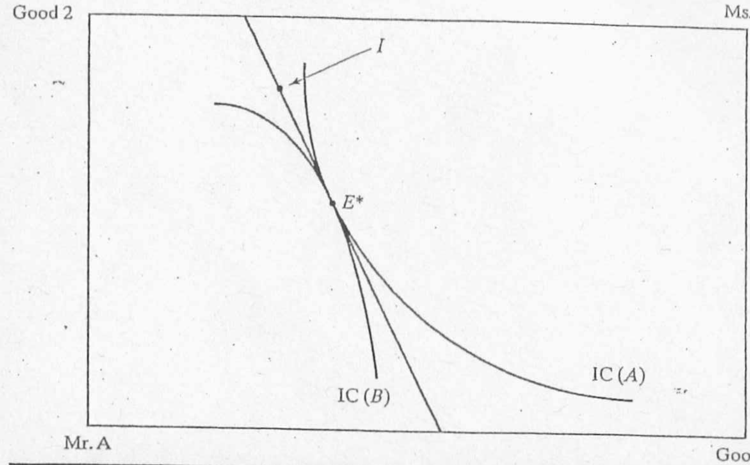
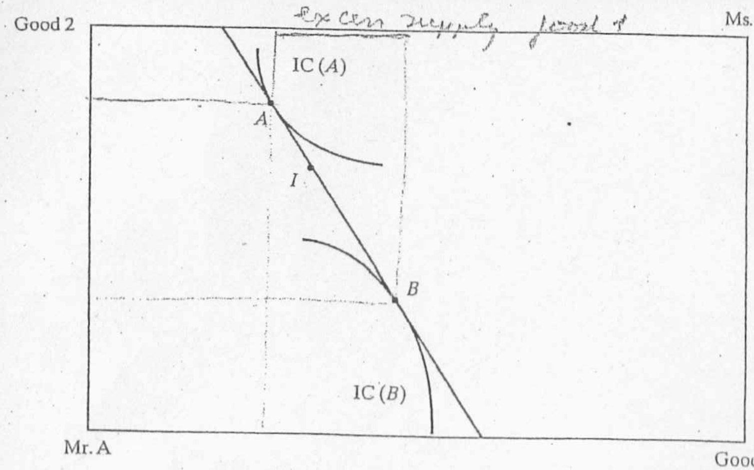


FIGURE A1-3 The Edgeworth-Bowley Box: Equilibrium Achieved at E^*

We will not say anything here about the conditions guaranteeing that such a process will converge. Let us rather insist on one crucial necessary precondition: that an equilibrium exists. In the text we have mentioned that assumptions H1 to H4 are needed to guarantee the existence of an equilibrium. Of course H4 does not apply here. H1 states the necessity of the existence of a price for each good, which is akin to specifying the existence of a price line. H2 defines one of the characteristics of a competitive equilibrium: that prices are taken as given by the various agents and the price line describes their perceived

FIGURE A1-4 The Edgeworth-Bowley Box: Disequilibrium, Excess Demand for Good 2, Excess Supply for Good 1



opportunity sets. Our discussion here can enlighten the need for H3. Indeed, in order for an equilibrium to have a chance to exist, the geometry of Figure A1-3 makes clear that the shapes of the two agents' ICs are relevant. The price line must be able to separate the "better than" areas of the two agents' ICs passing through a same point—the candidate equilibrium allocation. The better than area is simply the area above a given IC. It represents all the allocations providing higher utility than those on the level curve. This separation by a price line is not generally possible if the ICs are not convex, in which case an equilibrium cannot be guaranteed to exist. The problem is illustrated in Figure A1-5.

Once a competitive equilibrium is observed to exist, which logically could be the case even if the conditions that guarantee existence are not met, the Pareto optimality of the resulting allocation is insured by H1 and H2 only. In substance this is because once the common price line at which markets clear exists, the very fact that agents optimize taking prices as given, leads them to a point of tangency between their highest IC and the common price line. At the resulting allocation, both MRS are equal to the same price line and, consequently, are identical. The conditions for Pareto optimality are thus fulfilled.

FIGURE A1-5 The Edgeworth-Bowley Box: Non-Convex Indifference Curves